Evaluate $\int_{e^2}^{\infty} \frac{\ln x}{x^2}\,dx$.

SCORE: _____ / 6 PTS

$= \lim_{N\to\infty} \int_{e^2}^{N} \frac{\ln x}{x^2}\,dx$ (1)

$= \lim_{N\to\infty} \left. \frac{-\ln x - 1}{x} \right|_{e^2}^{N}$ (2)

$= \lim_{N\to\infty} \left(\frac{-\ln N - 1}{N} - \frac{-3}{e^2} \right)$ (1)

$= \frac{3}{e^2}$ (1)

$\ln x$ (+) x^{-2}

x^{-1} $\quad -x^{-1}$

1 $\quad -x^{-2}$

0 $\quad x^{-1}$

$\lim_{N\to\infty} \frac{-\ln N - 1}{N} \quad \frac{-\infty}{\infty}$

$= \lim_{N\to\infty} \frac{-\frac{1}{N}}{1}$ (1)

$= 0$

Evaluate $\int \frac{x^4+8}{x^3-4x}\,dx$.

$$x^3-4x \overline{)\, x^4 \qquad\qquad +8}$$
$$x^4 \quad -4x^2$$
$$4x^2 \qquad +8$$

SCORE: ____ / 9 PTS

$$= \int \left(x + \frac{4x^2+8}{x(x+2)(x-2)}\right) dx \quad \left(\tfrac{1}{2}\right) \quad \left(\tfrac{1}{2}\right)$$

$$= \int \left(x + \boxed{\frac{2}{x}} + \boxed{\frac{3}{x+2}} + \boxed{\frac{3}{x-2}}\right) dx \quad (1) \quad (1) \quad (1)$$

$$= \frac{1}{2}x^2 \quad \left(\tfrac{1}{2}\right) - 2\ln|x| \quad \left(\tfrac{1}{2}\right) + 3\ln|x+2| \quad \left(\tfrac{1}{2}\right) + 3\ln|x-2| + C \quad \left(\tfrac{1}{2}\right)$$

$$\frac{4x^2+8}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2} \quad (1)$$

$$4x^2+8 = A(x+2)(x-2) + Bx(x-2) + Cx(x+2) \quad (1)$$

$x=0$: $8 = -4A \rightarrow A=-2$

$x=2$: $24 = 8C \rightarrow C=3$

$x=-2$: $24 = 8B \rightarrow B=3$

SANITY CHECK:

$x=3$: LHS $= \frac{81+8}{27-12} = \frac{89}{15}$

RHS $= 3 - \frac{2}{3} + \frac{3}{5} + \frac{3}{1}$

$= \frac{45-10+9+45}{15} = \frac{89}{15}$ ✓ (1)

NO PARTIAL CREDIT IF WRONG COEFFICIENTS AND NO/INCORRECT SANITY CHECK

TALK TO ME IF YOU USED OTHER VALUE OF X IN SANITY CHECK

Evaluate $\int \frac{52-x}{x^3+4x^2+13x}\,dx$. $= \int \frac{52-x}{x(x^2+4x+13)}\,dx$ $x^2+4x+13 = (x+2)^2+9$ **SCORE: ____ / 9 PTS**

$b^2-4ac = 16-52<0$

$= \int \left(\frac{A}{x} + \frac{B(2x+4)+C}{x^2+4x+13}\right) dx$ (1)

$A(x^2+4x+13) + Bx(2x+4) + Cx = 52-x$ (1)

$x=0$: $13A = 52 \rightarrow A = 4$

$x=-2$: $9A - 2C = 54 \rightarrow -2C = 18 \rightarrow C = -9$

x^2: $A + 2B = 0 \rightarrow B = -\frac{1}{2}A \rightarrow B = -2$

$= \int \left(\underbrace{\frac{4}{x}}_{(1)} + \frac{\overbrace{-2(2x+4)}^{(1)} + 9}{x^2+4x+13}\right)^{(1)} dx$

$= \underbrace{4\ln|x|}_{(\frac{1}{2})} - \underbrace{2\ln(x^2+4x+13)}_{(1)} \underbrace{-3\tan^{-1}\frac{x+2}{3}}_{(1)} + C$ (1)

SANITY CHECK:

$x=2$: $LHS = \frac{50}{8+16+26} = \frac{50}{50} = 1$

$RHS = \frac{4}{2} + \frac{-2(8)-9}{4+8+13} = 2 - \frac{25}{25} = 1$

($\frac{1}{2}$)

NO PARTIAL CREDIT IF WRONG COEFFICIENTS AND NO/INCORRECT SANITY CHECK

TALK TO ME IF YOU USED OTHER VALUE OF X FOR SANITY CHECK

Evaluate $\int_0^8 \frac{1}{x+\sqrt[3]{x}}\,dx$.

SCORE: ____ / 6 PTS

$= \lim_{N\to 0^+} \int_N^8 \frac{1}{x+\sqrt[3]{x}}\,dx$ (1)

$= \lim_{N\to 0^+} \frac{3}{2}\ln(x^{\frac{2}{3}}+1)\Big|_N^8$ (1)

$= \lim_{N\to 0^+} \frac{3}{2}(\ln 5 - \ln(N^{\frac{2}{3}}+1))$ (1)

$= \frac{3}{2}\ln 5$ (1)

$\int \frac{1}{x+\sqrt[3]{x}}\,dx$

$= \int \frac{1}{\sqrt[3]{x}} \cdot \frac{1}{x^{\frac{2}{3}}+1}\,dx$

$= \frac{3}{2}\int \frac{1}{u}\,du$ (1)

$= \frac{3}{2}\ln|u| + C$

$= \frac{3}{2}\ln(x^{\frac{2}{3}}+1) + C$

$u = x^{\frac{2}{3}}+1$ (1)

$du = \frac{2}{3}x^{-\frac{1}{3}}\,dx$

$\frac{3}{2}du = \frac{1}{\sqrt[3]{x}}\,dx$

SEE ALSO ALTERNATE SOLUTION BELOW — GRADE AGAINST ONLY 1 VERSION OF SOLUTION

Evaluate $\int_0^8 \frac{1}{x+\sqrt[3]{x}}\,dx$.

SCORE: ____ / 6 PTS

$= \lim_{N\to 0^+} \int_N^8 \frac{1}{x+\sqrt[3]{x}}\,dx$ (1)

$= \lim_{N\to 0^+} \frac{3}{2}\ln(x^{\frac{2}{3}}+1)\Big|_N^8$ (1)

$= \lim_{N\to 0^+} \frac{3}{2}(\ln 5 - \ln(N^{\frac{2}{3}}+1))$ (1)

$= \frac{3}{2}\ln 5$ (1)

$u = \sqrt[3]{x}$ (1/2)

$x = u^3$

$dx = 3u^2\,du$

$\int \frac{1}{u^3+u} 3u^2\,du = \int \frac{3u}{u^2+1}\,du$ (1/2)

$t = u^2+1$ (1/2)

$dt = 2u\,du$

$\frac{3}{2}dt = 3u\,du$

$\frac{3}{2}\int \frac{1}{t}\,dt = \frac{3}{2}\ln|t|+C$ (1/2)

$= \frac{3}{2}\ln(u^2+1)+C$

$= \frac{3}{2}\ln(x^{\frac{2}{3}}+1)+C$